Elementary Analysis: The Theory of Calculus by Ross Exercise Solutions

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Chapter 1: The Set $\mathbb{N}$ of Natural Numbers

1. Prove $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ for all positive integers n

   Solution:
   Base Case: $1 = \frac{1}{6}(1)(2)(3) = 1$ ✓
   Inductive Step: Assume that $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ is true for some $n \in \mathbb{N}$. Then,

$$\begin{aligned}
1^2 + 2^2 + \cdots + n^2 + (n+1)^2 &= \frac{1}{6}n(n+1)(2n+1) + (n+1)^2 \\
&= \frac{1}{6}n(n+1)(2n+1) + \frac{6(n+1)^2}{6} \\
&= \frac{1}{6}(n+1)[n(2n+1) + 6(n+1)] \\
&= \frac{1}{6}(n+1)[2n^2 + n + 6n + 6] \\
&= \frac{1}{6}(n+1)(n+1+1)(2(n+1)+1)
\end{aligned}$$

   Hence, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ for all $n \in \mathbb{N}$

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2. Prove $3 + 11 + \cdots + (8n-5) = 4n^2 - n$ for all positive integers n.

   Solution:
   Base Case: $3 = 4(1)^2 - 1 = 3$ ✓
   Inductive Step: Assume that $3 + 11 + \cdots + (8n-5) = 4n^2 - n$ for some $n \in \mathbb{N}$. Then,

$$\begin{aligned}
3 + 11 + \cdots + (8n-5) + (8(n+1)-)5 &= 4n^2 - n + (8(n+1) - 5) \\
&= 4n^2 + 7n + 3 \\
&= 4(n^2 + 2n + 1) - n - 1 \\
&= 4(n+1)^2 - (n+1)
\end{aligned}$$

   Hence, $3 + 11 + \cdots + (8n-5) = 4n^2 - n$ for all $n \in \mathbb{N}$

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3. Prove $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$ for all postive integers n

Solution:
Base Case: $1^3 = 1^2$ ✓

Inductive Step: Assume that $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$ for some $n \in \mathbb{N}$. Note that $(x + y)^2 = x^2 + 2xy + y^2$. Then,

$$\begin{aligned}((1+2+\cdots+n)+(n+1))^2 &= (1+2+\cdots+n)^2+2(1+2+\cdots+n)(n+1)+(n+1)^2\\ (\textit{inductive hypothesis}) &= 1^3+2^3+\cdots+n^3+(n+1)(2(1+2+\cdots+n)+(n+1))\\ &= 1^3+2^3+\cdots+n^3+(n+1)((n)(n+1)+(n+1))\\ &= 1^3+2^3+\cdots+n^3+(n+1)^2(n+1)\\ &= 1^3+2^3+\cdots+(n+1)^3\end{aligned}$$

Hence, $1^3+2^3+\cdots+n^3 = (1+2+\cdots+n)^2$ implies that $1^3+2^3+\cdots+n^3+(n+1)^3 = (1+2+\cdots+n+(n+1))^2$ and thus, the statement holds for all $n \in \mathbb{N}$

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4. **a.)** Guess the formula for $1+3+\cdots+(2n-1)$ by evaluating the sum for $n = 1, 2, 3,$ and 4 [For $n = 1$, the sum is simply 1].
b.) Prove your formula using mathematical induction.

Solution:
We note that the sums appear to be of the form n^2.
Base Case: $1 = 1^2$ ✓

Inductive Step: Assume that $1 + 3 + \cdots + (2n - 1) = n^2$ is true for some $n \in \mathbb{N}$. Then

$$\begin{aligned}1+3+\cdots+(2n-1)+(2(n+1)-1) &= n^2+(2(n+1)-1)\\ &= n^2+2n+1\\ &= (n+1)^2\end{aligned}$$

Hence, $1 + 3 + \cdots + (2n - 1) = n^2$ for all positive integers n.

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5. Prove $1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ for all positive integers n.

Solution:
Base Case: $1 + \frac{1}{2} = 2 - \frac{1}{2}$ ✓

Inductive Hypothesis: Assume that $1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ is true for some $n \in \mathbb{N}$. Then,

$$\begin{aligned}1+\frac{1}{2}+\cdots+\frac{1}{2^n}+\frac{1}{2^{n+1}} &= 2-\frac{1}{2^n}+\frac{1}{2^{n+1}}\\ &= 2+\frac{1}{2^n}\left(\frac{1}{2}-1\right)\\ &= 2+\frac{1}{2^n}\left(-\frac{1}{2}\right)\\ &= 2-\frac{1}{2^{n+1}}\end{aligned}$$

Hence, $1 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ for all $n \in \mathbb{N}$. ■

6. Prove $11^n - 4^n$ is divisible by 7 when n is a positive integer.

Solution:
Base Case: $11 - 4 = 7(1)$ ✓

Inductive Step: Assume that $11^n - 4^n$ is divisible by 7 for some $n \in \mathbb{N}$. Then

$$\begin{aligned} 11^{n+1} - 4^{n+1} &= 11(11^n) - 11 \cdot 4^n + 11 \cdot 4^n - 4(4^n) \\ \textit{(inductive hypothesis)} &= 11(7m) + 4^n(11-4), \text{for some } m \in \mathbb{N} \\ &= 11(7m) + 4^n(7) \\ &= 7(11m + 4^n) \end{aligned}$$

Hence, $11^{n+1} - 4^{n+1}$ is divisible by 7, which proves that $11^n - 4^n$ is divisible by 7 for all $n \in \mathbb{N}$ ■

7. Prove $7^n - 6n - 1$ is divisible by 36 for all positive integers n.

Solution:
Base case: $0|36$ ✓

Inductive step: Assume that $7^n - 6n - 1$ is divisible by 36 for some $n \in \mathbb{N}$. Then,

$$\begin{aligned} 7^{n+1} - 6(n+1) - 1 &= 7(7^n) - 7(6n) - 7 + 36n \\ &= 7(7^n - 6n - 1) + 36n \\ (\textit{inductive hypothesis}) &= 7(36m) + 36n, \text{ for some } m \in \mathbb{N} \\ &= 36(7m + n) \end{aligned}$$

Hence, $7^n - 6n - 1$ is divisible by 36 for all positive integers n. ■

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Chapter 2: The Set $\mathbb{Q}$ of Rational Numbers

1. Show that $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{24}$, and $\sqrt{31}$ are not rational numbers

Solution:
We use the Rational Zeros Theoerem

$x^2 - 3 = 0 \implies x = \pm 1, \pm 3$, none of which are solutions and thus, $\sqrt{3}$ is not rational.

Similarly for the rest.

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2. Show $\sqrt[3]{2}$, $\sqrt[7]{5}$, and $\sqrt[4]{13}$ are not rational numbers

Solution:
If $\sqrt[3]{2}$ is rational, $x^3 - 2 = 0 \implies x = \pm 1, \pm 2$, none of which are solutions. Thus, $\sqrt[3]{2}$ is not rational.

Similarly for the rest.

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3. Show that $\sqrt{2+\sqrt{2}}$ is not a rational number.

   Solution:
   By the Rational Zeros Theroem, if $\sqrt{2+\sqrt{2}}$ is rational, then $(x^2-2)^2 = 2$ has a rational solution. Note that if x is rational, then so is $x^2 - 2$ and this becomes similar to the proof of 2.

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4. Show $\sqrt[3]{5-\sqrt{3}}$ is not a rational number.

   Solution:
   Proof is similar to (2.3).

   $\blacksquare$

5. Show $[3+\sqrt{2}]^{\frac{2}{2}}$ is not a rational number

   Solution:
   $x^3 = (3+\sqrt{2})^2 \implies (x^3-13)^2 = 36(2)$ Note that if x is rational, then (x^3-13) is also rational. Proof resolves similar to (2.2).

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6. In connection with Example 6, discuss why $4-7b^2$ is rational if b is rational.

   Solution:
   If b is rational, then b can be written as $\frac{m}{n}$, $m, n \in \mathbb{Z}$. Then $4-7b^2$ can be written as $\frac{4n^2-7m^2}{n^2}$ where $p = 4n^2 - 7m^2$ and $q = n^2$, $p, q \in \mathbb{Z}$ since $\mathbb{Z}$ is closed under addition and multiplication.

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Chapter 3: The Set $\mathbb{R}$ of Real Numbers

1. **(a)** Which of the properties A1-A4, M1-M4, DL, O1-O5 fail for $\mathbb{N}$?

   (b) Which of these properties fail for $\mathbb{Z}$?

   Solution:
   (a) A3 and A4 fails for $\mathbb{N}$ because $0, -a \notin \mathbb{N}$ for all $a \in \mathbb{N}$. M4 fails because $a^{-1} \notin \mathbb{N}$ for all $a \geq 2$.

   (b) M4 fails for $\mathbb{Z}$ because $a^{-1} \notin \mathbb{Z}$ for $a \geq 2$.

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3. Prove iv. ($(-a)(-b) = ab$ for all a, b) and v. ($ac = bc$ and $c \neq 0$ imply $a = b$) of Theorem 3.1

   Solution:

   $$(-a)(-b) + (-ab) = (-a)(-b) + (-a)b = (-a)[(-b) + b] = (-a)(0) = 0 = ab + (-ab)$$

   . From (i), this implies that $(-a)(-b) = ab$.

   $$a \overset{M3}{=} a \cdot 1 \overset{M4}{=} acc^{-1} \overset{M1}{=} (ac)c^{-1} \overset{hypothesis}{=} (bc)c^{-1} \overset{M1}{=} bcc^{-1} \overset{M4}{=} b \cdot 1 \overset{M3}{=} b$$

   $\blacksquare$

4. Prove v. and vii. of Theorem 3.2

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6. (a) Prove $|a+b+c| \leq |a|+|b|+|c|$ for all $a, b, c \in \mathbb{R}$. Hint: Apply the triangle inequality twice. Do not consider eight cases.

   (b) Use induction to prove

$$|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| \cdots + |a_n|$$

   Solution:
   (a)

$$\begin{aligned} |a+b+c| &\leq |a+b|+|c| && \textit{(triangle inequality)} \\ &\leq |a|+|b|+|c| && \textit{(triangle inequality)} \end{aligned}$$

   (b) Base Case: $|a_1| \leq |a_1| \checkmark$
   Inductive Step: Assume that $|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| \cdots + |a_n|$ is true for some $n \in \mathbb{N}$. Then,

$$\begin{aligned} |a_1 + a_2 + \cdots + a_n + a_{n+1}| &\leq |a_1 + a_2 + \cdots + a_n| + |a_{n+1}| && \textit{(triangle inequality)} \\ &\leq |a_1| + |a_2| \cdots + |a_n| + |a_{n+1}| && \textit{(inductive hypothesis)} \end{aligned}$$

   By induction, this proves that $|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| \cdots + |a_n|$ for all $n \in \mathbb{N}$.

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8. Let $a, b \in \mathbb{R}$. Show if $a \leq b_1$ for every $b_1 > b$, then $a \leq b$.

   Solution:
   Assume towards contradiction that $a > b$. Then $a - b > 0$. Set $b_1 = b + \frac{1}{2}(a-b)$ so then $b_1 > b$ for every value of $b \in \mathbb{R}$. Then $a - b_1 = a - b - \frac{1}{2}(a-b) = \frac{1}{2}(a-b) > 0$. Then $a < b_1$, which contradicts the fact that $a \leq b_1$. Hence, our assumption is invalid and $a \leq b$.

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Chapter 4: The Completeness Axiom

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5. Let S be a nonempty subset of $\mathbb{R}$ that is bounded above. Prove if $\sup S$ belongs to S, then $\sup S = \max S$. *Hint:* Your proof should be very short.

   Solution:
   Since $\sup S \geq s_0$ for all $s_0 \in S$ and $\sup S \in S$, by definition, $\sup S = \max S$.

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6. Let S be a nonempty bounded subset of R.

   (a) Prove inf $S \leq$ sup S. Hint: This is almost obvious; your proof should be short.

   (b) What can you say about S if inf $S =$ sup S?

   Solution:
   (a) We have that for any $s_0 \in S$, inf $S \leq s_0$ and $s_0 \leq$ sup S. Hence, inf $S \leq s_0 \leq$ sup S for all $s_0 \in S$ implies that inf $S \leq$ sup S

   (b) If inf $S =$ sup S, then from (a), we see that inf $S \leq s_0 \leq$ sup $S \implies$ inf $S = s_0 =$ sup S for all $s_0 \in S$. Hence, S contains only one element, namely s_0.

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7. Let S and T be nonempty bounded subsets of $\mathbb{R}$.

   (a) Prove if $S \subseteq T$, then inf $T \leq$ inf $S \leq$ sup $S \leq$ sup T .

   (b) Prove $\sup(S \cup T) = \max\{\sup S, \sup T\}$. Note: In part (b), do not assume $S \subseteq T$

8. Let S and T be nonempty subsets of R with the following property:

   $$s \leq t \text{ for all } s \in S \text{ and } t \in T .$$

   (a) Observe S is bounded above and T is bounded below.

   (b) Prove sup $S \leq$ inf T .

   (c) Give an example of such sets S and T where $S \cap T$ is nonempty.

   (d) Give an example of sets S and T where sup $S =$ inf T and $S \cap T$ is the empty set.

   Solution:
   (a) S is bounded above by T and T is bounded below by S. (b) We have that $s \leq$ sup S and inf $T \leq t$ for all $s \in S, t \in T$ Since $s \leq$ ■

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10. Prove that if $a > 0$, then there exists $n \in N$ such that $\frac{1}{n} < a < n$.

11. Consider $a, b \in R$ where $a < b$. Use Denseness of $\mathbb{Q}$ to show there are infinitely many rationals between a and b.

12. Let $\mathbb{I}$ be the set of real numbers that are not rational; elements of $\mathbb{I}$ are called *irrational numbers*. Prove if $a < b$, then there exists $x \in \mathbb{I}$ such that $a < x < b$. *Hint:* First show $\{r + \sqrt{2} : r \in \mathbb{Q}\} \subseteq \mathbb{I}$.

    Solution:
    First we show that $\{r + \sqrt{2} : r \in \mathbb{Q}\} \subseteq \mathbb{I}$. Let $x = r + \sqrt{2}$ and assume $x \in \mathbb{Q}$. Since $-r \in \mathbb{Q}$, we have that $x + (-r) = \sqrt{2}$. If x is rational, then $x + (-r)$ should also be rational. But we've already seen that $\sqrt{2}$ is not rational. Hence, our original assumtion that x is rational was false and $\{r + \sqrt{2} : r \in \mathbb{Q}\}$.

    If $a < b$, then $a + \sqrt{2} < b + \sqrt{2}$ and by the denseness of $\mathbb{Q} in \mathbb{R}$, we have that there exists a $r \in \mathbb{Q}$ such that $a + \sqrt{2} < r < b + \sqrt{2}$. This implies $a < r - \sqrt{2} < b$. Suppose $r - \sqrt{2}$ is rational. Then by the above, we have that $r - \sqrt{2} + \sqrt{2} = r \in \mathbb{I}$ This is a contradiction, so $r - \sqrt{2}$ must be irrational. Then let $x = r - \sqrt{2}$ so that $a < x < b$, as desired.

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14. Let A and B be nonempty bounded subsets of $\mathbb{R}$, and let $A+B$ be the set of all sums $a+b$ where $a \in A$ and $b \in B$.

(a) Prove $\sup(A+B) = \sup A + \sup B$. *Hint:* To show $\sup A + \sup B \leq \sup(A+B)$, show that for each $b \in B$, $\sup(A+B) - b$ is an upper bound for A, hence $\sup A \leq \sup(A+B) - b$. Then show $\sup(A+B) - \sup A$ is an upper bound for B.

(b) Prove $\inf(A+B) = \inf A + \inf B$.

Solution:
We have that $a \leq \sup A$ for all $a \in A$ and that $b \leq \sup B$ for all $b \in B$. Then $(a+b) \leq \sup A + \sup B$ and since $\sup(A+B)$ is the least upper bound, then $\sup(A+B) \leq \sup A + \sup B$.

Since $a+b \leq \sup(A+B)$, then $a \leq \sup(A+B) - b$ for each $b \in B$. We have found an upper bound for $A4$ and hence, $\sup A \leq \sup(A+B) - b$. This holds for any arbitrary $b \in B$ and so $\sup(A+B) - \sup A$ is an upper bound for b. Hence, $\sup B \leq \sup(A+B) - \sup A$. Thus, $\sup A + \sup B \leq \sup(A+B)$ and we have shown that $\sup(A+B) = \sup A + \sup B$ ■

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Chapter 5: The Symbols $+\infty$ and $-\infty$

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Chapter 7: Limits of Sequences

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Chapter 8: A Discussion About Proofs

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Chapter 9: Limit Theorems for Sequences

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Chapter 10: Monotone Sequences and Cauchy Sequences

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Chapter 11: Subsequences

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Chapter 12: lim sup's and lim inf's

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Chapter 14: Series

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Chapter 15: Alternating Series and Integral Tests

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Chapter 17: Continuous Functions

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Chapter 18: Properties of Continuous Functions

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Chapter 19: Uniform Continuity

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Chapter 20: Limits of Functions

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Chapter 21: More on Metric Spaces: Continuity

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Chapter 22: More on Metric Spaces: Connectedness

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Chapter 23: Power Series

1. For each of the following power series, find the radius of convergenceand determine the exact interval of convergence.

   (a)

2. Repeat Exercise 23.1 for the following:

   (a) $\sum \sqrt{n} x^n$
   (b) $\sum \frac{1}{n\sqrt{n}} x^n$
   (c) $\sum x^{n!}$
   (d) $\sum \frac{3^n}{\sqrt{n}} x^{2n+1}$

   Solution:

   (a) We have $a_n = \sqrt{n}$ so then $\limsup |a_n|^1/n = \limsup |n^{\frac{1}{2n}}| = 1$. Thus, $\beta = 1 \implies R = 1$. The radius of convergence is 1.
   For $x = -1$, we have $\sum(-1)^n \sqrt{n}$ which diverges since $\lim(-1)^n \sqrt{n} \neq 0$.
   For $x = 1$, we have $\sum \sqrt{n}$ which also diverges since $\lim \sqrt{n} \neq 0$.
   Thus, the interval of convergence is $(-1, 1)$.

(b) We have $a_n = \frac{1}{n^{\sqrt{n}}}$ so let $\beta = \limsup \left|\frac{1}{n^{n^{1/2}}}\right|^{1/n} = \limsup \left|\frac{1}{n^{\frac{1}{\sqrt{n}}}}\right| = 1$. Thus, $R = 1$ and the radius of convergence is 1.

For $x = -1$, we have $\sum (-1)^n \frac{1}{n^{\sqrt{n}}}$. Since $a_n > a_{n+1}$ $\forall n \in \mathbb{N}$ and $\lim a_n = 0$, then by the alternating series test, this series converges.

For $x = 1$, we have $\sum \frac{1}{n^{\sqrt{n}}}$. Since $\frac{1}{n^{\sqrt{n}}} \leq \frac{1}{n^2}$ for $n \geq 4$ and $\sum \frac{1}{n^2}$ converges by Theorem 15.1, then $\sum \frac{1}{n^{\sqrt{n}}}$ converges by the comparison test.

Thus, the interval of convergence is $[-1, 1]$.

(c) $\big(x^1 + x^2 + x^6 + \dots\big)$.

We have $a_n = \begin{cases} 0 & \text{if } n \text{ is not the result of an integer factorial} \\ 1 & \text{if } n = k! \text{ for some } k \in \mathbb{N} \cup \{0\} \end{cases}$

Let $\beta = \limsup |a_n|^{1/n} = \limsup |a_{k!}|^{\frac{1}{k!}} = \limsup (1)^{\frac{1}{k!}} = 1$.

For $x = -1$, $\sum (-1)^{n!}$ diverges since $\lim (-1)^{n!} \neq 0$.
For $x = 1$, $\sum 1^{n!}$ diverges since $\lim 1^{n!} \neq 0$.
Thus, the interval of convergence is $(-1, 1)$.

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Chapter 24: Uniform Convergence

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2. For $x \in [0, \infty)$, let $f_n(x) = \frac{x}{n}$.

   (a) Find $f(x) = \lim f_n(x)$.

   (b) Determine whether $f_n \to f$ uniformly on $[0, 1]$.

   (c) Determine whether $f_n \to f$ uniformly on $[0, \infty)$.

   Solution:

   (a) We have $\lim f_n(x) = \lim \frac{x}{n} = 0$ for $x \in [0, \infty)$.

(b) Let $N = \frac{1}{\epsilon}$ so that for all $n > N, \epsilon > 0$ we have that

$$|f_n(x) - f(x)| = \left|\frac{x}{n} - 0\right| = \frac{x}{n} < \frac{1}{\frac{1}{\epsilon}} = \epsilon$$

for all $x \in [0,1]$. Hence, $f_n \to f$ uniformly on $[0,1]$

(c) Suppose by contradiction that $f_n \to f$ uniformly on $[0,\infty)$. Then for $\epsilon = 1$, there exists N such that $|f_n(x) - f(x)| < 1$ for all $x \in [0,\infty)$ and $n > N$. This implies $|f_n(x) - f(x)| = \frac{x}{n} < 1$ for all $x \in [0,\infty)$. But for $x = 2n \in [0,\infty)$, we have that

$$|f_n(x) - f(x)| = \frac{x}{n} = \frac{2n}{n} = 2 > 1$$

a contradiction. Hence, f_n does not converge uniformly to f on $[0,\infty)$.

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Chapter 25: More on Uniform Convergence

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Chapter 26: Differentiation and Integration of Power Series

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Chapter 27: Weierstrass's Approximation Theorem

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Chapter 28: Basic Properties of the Derivative

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Chapter 29: The Mean Value Theorem

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Chapter 30: L'Hospital's Rule

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Chapter 31: Taylor's Theorem

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Chapter 32: The Riemann Integral

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Chapter 33: Properties of the Riemann Integral

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Chapter 34: Fundamental Theorem of Calculus

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Chapter 35: Riemann-Stieltjes Integrals

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Chapter 36: Improper Integrals

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Chapter 37: A Discussion of Exponents and Logarithms

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Chapter 38: Continuous Nowhere-Differentiable Functions

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